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Abstract

This paper shows how and when it is possible to obtain a mapping from a quarterly dynamic stochastic general equilibrium (DSGE) model to a monthly specification that maintains the same economic restrictions and has real coefficients. We use this technique to derive the monthly counterpart of the well-known DSGE model by Galí, Smets and Wouters (GSW) for the US economy. We then augment it with auxiliary macro indicators which, because of their timeliness, can be used to obtain a nowcast of the structural model. We show empirical results for the quarterly growth rate of GDP, the monthly unemployment rate and GSW's welfare-relevant output gap. Results show that the augmented monthly model does best for nowcasting.

Key words: Forecasting, temporal aggregation, mixed frequency data, large data sets.

JEL classification: C33, C53, E30.

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Summary

Central bankers typically place their forecasts centre-stage in the communications. The forecast is presented as a guide to future developments in the economy, but also as a means of communicating policy. Thus statistical models that might forecast well but have no economic interpretation are insufficient to meet the policy explanation task. Yet to deliver accurate predictions, especially in the short term, it is vital that the forecast incorporates a range – often a very wide range – of timely information.

Consequently the preparation of a forecast is a subtle process of aggregation of knowledge of how the economy and policy channels work and detailed data. This process involves the combination of formal models which aid understanding, expert judgement and statistical data analysis. In this paper we address a particular part of this process and analyse the connection between two important tools in the forecasting process: the structural quarterly model and the daily monitoring of monthly data releases for the assessment of the current state of the economy.

The quarterly structural model is often a dynamic stochastic general equilibrium (DSGE) model which looks at the joint evolution of the key relations in the economy as it is buffeted by random (stochastic) shocks and is deeply driven by economic theory. It is essential when constructing scenarios based on different policy paths or other conditioning assumptions; that is, for policy analysis. The objective of this is not to obtain a simple forecast, but rather to analyse the implications of policy alternatives. Moreover, from structural models one can recover quantities that are not directly observable from the data but that are often relevant for the understanding of the stance of policy, such as the natural rate of interest or potential output. Although this part of the analysis is essential for guiding the policy discussion, any decision maker needs to have, in addition, a system in place for understanding the evolution of the current state of the economy. Such a system involves the analysis of many different items of data, including surveys or conjunctural leading indicators which are published early in the quarter, before the release of the quarterly national accounts data that the DSGE model is largely designed to explain, and can provide a timely signal on quantities of key interest such as GDP or employment. For this function, the typical structural model is of no use since it is not designed to capture realistic features of the data flow: a non-synchronous calendar of publications, mixed frequency (meaning a mixture of quarterly, monthly or even daily data), and potentially a very large dimension, with perhaps hundreds of data series. But recent work has developed a statistical framework for dealing with these problems, allowing continuous updates of the estimate of the current state of the economy in relation to the real time data flow. This process is sometimes labelled nowcasting, the point being it is helpful in very short-run forecasts.

This paper proposes a framework that bridges a structural quarterly model and a statistical model for nowcasting. This is particularly relevant for the conduct of monetary policy today when, with the implementation of forward guidance, an increasing emphasis has been placed on the definition and communication of nearer term policy in relation to the evolution of the state of the economy. With our methodology it is possible to exploit the real data flow in a DSGE model, using the most recent data for both the variables that appear in the model and other series that are not explicitly modelled, but that might be informative about the current state of the economy. Because this information is processed through the structural model, we can update the policy analysis at each new data release and assess the impact of the new piece of information on our scenarios and on our estimates of policy relevant and inherently model-dependent concepts, such as the equilibrium real interest rate or potential output.

We apply our methodology to a medium-scale DSGE model and to a sizable number of timely macroeconomics series for the United States and we show that we obtain point and density nowcasts that are much more accurate than those obtained with the quarterly model, and

comparable with a traditionally tough benchmark, such as the Survey of Professional Forecasters' nowcasts.

1 Introduction

Central bankers, including the Bank of England, typically place their forecasts centre-stage in the communications. The forecast is presented as a guide to future developments in the economy (for example to inflation and growth), but also as a means of communicating policy. Thus statistical models that might forecast well but have no economic interpretation are insufficient to meet the policy explanation task. Yet to deliver accurate predictions, especially in the short term, it is vital that the forecast incorporates a range - often a very wide range - of timely information.

Consequently the preparation of a forecast is a subtle process of aggregation of knowledge of how the economy and policy channels work and detailed data. This process involves the combination of formal models which aid understanding, expert judgement and statistical data analysis. In this paper we address a particular part of this process and analyse the connection between two important tools in the forecasting process: the structural quarterly model and the daily monitoring of monthly data releases for the assessment of the current state of the economy. The quarterly structural model is often a dynamic stochastic general equilibrium (DSGE) model which looks at the joint evolution of the key relations in the economy as it is buffeted by random (stochastic) shocks and is deeply driven by economic theory. It is essential when constructing scenarios based on different policy paths or other conditioning assumptions; that is, for policy analysis. The objective of this is not to obtain a simple forecast, but rather to analyse the implications of policy alternatives. Moreover, from structural models one can recover quantities that are not directly observable from the data but that are often relevant for the understanding of the stance of policy, such as the natural rate of interest or potential output. Although this part of the analysis is essential for guiding the policy discussion, any decision maker needs to have, in addition, a system in place for understanding the evolution of the current state of the economy. Such a system involves the analysis of many different items of data, including surveys or conjunctural leading indicators which are published early in the quarter, before the release of the quarterly national accounts data that the DSGE model is largely designed to explain, and can provide a timely signal on quantities of key interest such as GDP or employment. For this function, the typical structural model is of no use since it is not designed to capture realistic features of the data flow: a non-synchronous calendar of publications, mixed frequency (meaning a mixture of quarterly, monthly or even daily data), and potentially a very large dimension, with perhaps hundreds of data series. But recent work has developed a statistical framework for dealing with these problems, allowing continuous updates of the estimate of the current state of the economy in relation to the real time data flow. This process is sometimes labelled nowcasting, the point being it is helpful in very short-run forecasts.

This paper proposes a framework that bridges a structural quarterly model and a statistical model for nowcasting. This is particularly relevant for the conduct of monetary policy today when, with the implementation of forward guidance, an increasing emphasis has been placed on the definition and communication of nearer term policy in relation to the evolution of the state of the economy.

We build on recent work by Giannone, Monti and Reichlin (2010) and provide a mapping between a quarterly DSGE model and a nowcasting monthly model. The first contribution of this paper is to provide a way of (i) assessing when a linear or linearized quarterly model has a unique monthly specification with real coefficients and (ii) selecting the appropriate monthly specification, if there is more than one. The second contribution is to provide an analytical

framework for augmenting the monthly model with timely auxiliary variables which can be exploited to obtain early estimates of the variables of interest each time data are released according to the publication calendar. The DSGE model we use for our analysis is Galí, Smets and Wouters (2012) and we augment it with fifteen relevant macroeconomic variables.

The method we propose achieves three objectives. First, it allows to derive the monthly dynamics of the model, addressing a classic problem of time aggregation (for an early discussion, see Hansen and Sargent 1991). Second, it exploits the additional information in the auxiliary data in an analytically consistent way. Third, it allows us to exploit the timeliness of the auxiliary variables by incorporating them in the model as they become available according to the real time calendar. As mentioned, beside nowcasting, our framework produces model-based estimates of economically important unobservable variables. In our application we analyse the nowcast of the DSGE based output gap constructed from the flexible price equilibrium path.

Our work is related to two strands of the literature. The first is a recent set of works which use mixed frequency data to improve the estimation of the structural parameters of a quarterly DSGE model, by alleviating the temporal aggregation bias and mitigating identification issues (see Foroni and Marcellino, 2013, and Kim, 2010). The second, is Boivin and Giannoni (2006) which have proposed to augment a structural DSGE model by quarterly auxiliary variables in order to improve estimation of the quarterly structural parameters. Contrary to this literature, our emphasis is not on estimation or identification but on obtaining a timely estimate of GDP, unemployment and output gap, given the estimates of the parameters of the quarterly DSGE model.

2 The methodology

We consider structural quarterly models whose log-linearized solution has the form:

$$
\begin{aligned}
s_{t_q} &= \mathcal{T}_\theta s_{t_q-1} + \mathcal{B}_\theta \varepsilon_{t_q} \\
Y_{t_q} &= \mathcal{M}_{0,\theta} s_{t_q} + \mathcal{M}_{1,\theta} s_{t_q-1}
\end{aligned}
\tag{1}
$$

where t_q is time in quarters, $Y_{t_q} = (y_{1,t_q}, ..., y_{k,t_q})'$ is a set of observable variables which are transformed to be stationary, s_t are the states of the model and ε_t are structural orthonormal shocks. The autoregressive matrix $\mathcal{T}_\theta$, the coefficients $\mathcal{B}_\theta$, $\mathcal{M}_{0,\theta}$ and $\mathcal{M}_{1,\theta}$ are function of the deep, behavioural parameters of the DSGE model, which are collected in the vector θ. $\mathcal{M}_{1,\theta}$ accounts for the fact that often a part of the observables are defined in first differences. We will consider the model and its parameters as given. The vector s_t can also include the lags of the state variables and shocks.

In what follows, we will show how to obtain the monthly specification of the quarterly DSGE model that has real coefficients and we will discuss under which conditions such a monthly model exists and is unique. We will then discuss how to link the monthly model with the auxiliary variables for nowcasting.

2.1 From monthly to quarterly specification

Let us now define t_m as the time in months and denote by $Y_{t_m} = (y_{1,t_m}, ..., y_{k,t_m})'$ the vector of the possibly latent monthly counterparts of the variables that enter the quarterly model. The latter are transformed so as to correspond to a quarterly quantity when observed at end

of the quarter, i.e. when t_m corresponds to March, June, September or December (e.g. see Giannone *et al.*, 2008).

For example, let y_{i,t_m} be the unemployment rate u_{t_m} and suppose that it enters the quarterly model as an average over the quarter, then:

$$y_{i,t_m} = \frac{1}{3}(u_{t_m} + u_{t_m-1} + u_{t_m-2})$$

In accordance with our definition of the monthly variables, we can define the vector of monthly states s_{t_m} as a set of latent variables which corresponds to its quarterly model-based concept when observed on the last month of each quarter. Hence, it follows that our original state equation

$$s_{t_q} = \mathcal{T}_\theta \, s_{t_q-1} + B_\theta \varepsilon_{t_q}$$

can be rewritten in terms of the monthly latent states as

$$s_{t_m} = \mathcal{T}_\theta \, s_{t_m-3} + B_\theta \varepsilon_{t_m} \tag{2}$$

when t_m corresponds to the last month of a quarter, i.e. when t_m corresponds to March, June, September or December.

We will assume that the monthly states can be written as

$$s_{t_m} = \mathcal{T}_m \, s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m} \tag{3}$$

and we assume that $\mathcal{T}_m$ is real and stable and ε_{m,t_m} are orthonormal shocks[1]. This implies:

$$s_{t_m} = \mathcal{T}_m^3 \, s_{t_m-3} + [\mathcal{B}_m \varepsilon_{m,t_m} + \mathcal{T}_m \mathcal{B}_m \varepsilon_{m,t_m-1} + \mathcal{T}_m^2 \mathcal{B}_m \varepsilon_{m,t_m-2}]. \tag{4}$$

We are interested in finding a mapping from the quarterly model to the monthly model: the relation between equations (1), or equivalently (2), and (4) imply that the monthly model can be recovered from the following equations.

$$
\begin{aligned}
\mathcal{T}_m &= \mathcal{T}_\theta^{\frac{1}{3}} \\
vec(\mathcal{B}_m \mathcal{B}_m') &= (I + \mathcal{T}_m \otimes \mathcal{T}_m + \mathcal{T}_m^2 \otimes \mathcal{T}_m^2)^{-1} vec(\mathcal{B}_\theta \mathcal{B}_\theta').
\end{aligned}
\tag{5}
$$

From (5) it is clear that finding such mapping is equivalent to finding the cube root of $\mathcal{T}_\theta$.

If the autoregressive matrix of the transition equation is diagonalizable, *i.e* if there exist a diagonal matrix D and an invertible matrix V such that $\mathcal{T}_\theta = VDV^{-1}$, then the cube root of $\mathcal{T}_\theta$ can be obtained as

$$\mathcal{T}_\theta^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1},$$

where $D^{\frac{1}{3}}$ is a diagonal matrix containing the cube roots of the elements of D. We are looking for the real-valued $\mathcal{T}_\theta^{\frac{1}{3}}$ and we proceed as follows. The real elements of D, which are associated with real-valued eigenvectors, have a unique real cube root, and the latter is the only one that gives rise to real coefficients. Therefore, in the case of real eigenvalues, we simply select their real cube root. For the eigenvalues that are complex conjugate instead there are three

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[1] If the variables considered are stocks, the formulation (3) implies no approximation, because selecting a lower frequency just means sampling at a different frequency. If instead the variables considered are flows, then our definition of the monthly variables as an average over the quarter implies that we are introducing a non-invertible moving average in the growth rates. Therefore modeling this monthly concept as autoregressive introduces some misspecification, but Doz et al. (2012) show the effect of such misspecification is small.

complex cube roots. To select among these alternative roots, we choose the cube root which is characterized by less oscillatory behaviour, i.e. the cube root with smaller argument. When $\mathcal{T}_\theta$ is diagonalizable, it is also possible to characterize all the cube roots of the matrix, and verify which, if any, have real coefficients (for an example, see Appendix B, where we discuss how to do so using a prototypical New Keynesian model). Moreover, if monthly observations for some variables are available, we can use them to identify the cube root by choosing the one that maximizes the likelihood of the data. The cube root selected is generally unique. Indeed, Anderson et al. (2014) have shown that having mixed frequency observation typically implies identifiability. In our case the two procedures produce the same results.

If $\mathcal{T}_\theta$ is not diagonalizable, it is possible to obtain the Jordan form[2] and to obtain the cube root based on that. An interesting result is that the procedure described for diagonalizable matrices extends to this situation in most cases (see Higham, 2008). However there is a caveat that is of particular relevance for DSGE models. Namely, Higham (2008) proves that there exists no p-th (so also no cube) root of a matrix that has zero-valued eigenvalues that are defective, *i.e.* that are multiple but not associated to linearly independent eigenvectors. In the case of DSGE models, this situation arise mainly, but not exclusively, when there are redundant states. It is hence important to work on the model to try to reduce it to a minimal state space. When defective zero-valued eigenvalues appear even in the transition matrix of the minimal state space [3], then we suggest considering whether there are ways to render the model diagonalizable (see the example in the following Section).

Let us now turn to the equation that links the states to the observables. We will start by analyzing the (not very realistic) case in which all variables are observable at monthly frequency. The monthly observation equation would then be:

$$Y_{t_m} = \mathcal{M}_m s_{t_m} \tag{6}$$

where

$$\mathcal{M}_m = \left(\mathcal{M}_{0,\theta} + 0 \cdot L + 0 \cdot L^2 + \mathcal{M}_{1,\theta} L^3 \right)$$

The equations (3) and (6) therefore describe the monthly dynamics that are compatible with the quarterly model. If all the observables of the model were available at a monthly frequency, we could now simply use the monthly model defined by equations (3) and (6) to immediately incorporate this higher frequency information. However, some variables - think of GDP, for example - are not available at monthly frequency. So let us assume that the variable in the i-th position of the vector of observables Y_{t_m}, i.e. y_{i,t_m}, is not available at a monthly frequency, but only at the quarterly frequency. This means that y_{i,t_m} is a latent variable when t_m does not correspond to the end of a quarter. Moreover, due to the unsynchronized data

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[2] Any matrix $A \in \mathbb{C}^{n \times n}$ can be expressed in the canonical Jordan form

$$Z^{-1} A Z = J = diag(J_1, J_2, ..., J_p),$$

with

$$J_k = J_k(\lambda_k) = \begin{bmatrix} \lambda_k & 1 & & \\ & \lambda_k & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_k \end{bmatrix} \in \mathbb{C}^{m_k \times m_k},$$

where Z is non-singular and $m_1 + m_2 + + ... + m_p = n$ with p the number of blocks. We will denote by s the number of distinct eigenvalues (see, for example, Higham (2008) for further details).

[3] For example because of the choice of observables

releases schedule data are not available on the same span (the dataset has jagged edges). The unavailability of some data does not prevent us from still taking advantage of the monthly information that is available using a Kalman filter. To do so, we follow Giannone, Reichlin and Small (2008) and define the following state space model

$$s_{t_m} = \mathcal{T}_m \, s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m}$$
$$Y_{t_m} = \mathcal{M}_m(L) s_{t_m} + V_{t_m}$$

where $V_{t_m} = (v_{1,t_m}, ..., v_{k,t_m})$ is such that $\mathrm{var}(v_{i,t_m}) = 0$ if y_{i,t_m} is available and $\mathrm{var}(v_{i,t_m}) = \infty$ otherwise.

2.2   Bridging the model with the additional information

Let us now discuss how we bridge the model with additional monthly variables that carry information on current economic conditions. We define by $X_{t_m} = (x_{1,t}, ..., x_{n,t})'$ the vector of these auxiliary stationary monthly variables transformed so as to correspond to quarterly quantities at the end of each quarter.

For example, let us consider the index of capacity utilization CU_{t_m} and suppose that, to make it stationary, we have to take first differences. Then, assuming CU_{t_m} is in the j−th position of the vector of auxiliary variables, we have:

$$x_{j,t_m} = \frac{1}{3} \left[(CU_{t_m} + CU_{t_m-1} + CU_{t_m-2}) - (CU_{t_m-3} + CU_{t_m-4} + CU_{t_m-5}) \right] \tag{7}$$

which, when observed at the last month of a quarter, corresponds to the quarterly change of the average capacity utilization over that quarter.[4]

Let us now turn to how we incorporate the auxiliary monthly variables in the structural model. As a starting point we the define the relation between the auxiliary variables X_{t_q} and the model's observable variables at a quarterly frequency:

$$X_{t_q} = \mu + \Lambda Y_{t_q} + e_{t_q} \tag{8}$$

where e_{t_q} is orthogonal to the quarterly variables entering the model. Given that some of the observables are available only at a quarterly frequency, we will use this equation to estimate the coefficients Λ and the variance-covariance matrix of the shocks $E(e_{t_q} e_{t_q}') = R$. Let us now focus on incorporating the auxiliary variables in their monthly form. As stressed above, $X_{t_m} = (x_{1,t}, ..., x_{n,t})'$ is the vector of these auxiliary stationary monthly variables transformed so as to correspond to quarterly quantities at the end of each quarter. We can relate X_{t_m} to the monthly observables Y_{t_m} using the equivalent of equation (8) for the monthly frequency (the bridge model):

$$X_{t_m} = \mu + \Lambda Y_{t_m} + e_{t_m} \tag{9}$$

where $e_{t_m} = (e_{1,t_m}, ..., e_{k,t_m})$ is such that $\mathrm{var}(e_{i,t_m}) = [R]_{i,i}$ if X_{i,t_m} is available and $\mathrm{var}(e_{i,t_m}) = \infty$ otherwise. This way we take care of the problem of the jagged edge at the end of the dataset,

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[4]If capacity utilization is instead already stationary in the level then

$$x_{j,t_m} = \frac{1}{3}(CU_{t_m} + CU_{t_m-1} + CU_{t_m-2}))$$

which corresponds to the average capacity utilization over the quarter.

due to the fact that the data is released in an unsynchronized fashion and that the variables have different publishing lags (e.g. capacity utilization releases refer to the *previous* month's total capacity utilization, while the release of the Philadelphia Business Outlook Survey refers to the *current* month). We will use equation (9) to expand the original state-space:

$$
\begin{aligned}
s_{t_m} &= \mathcal{T}_m\, s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m} \\
Y_{t_m} &= \mathcal{M}_m(L) s_{t_m} + V_{t_m} \\
X_{t_m} - \mu &= \Lambda Y_{t_m} + e_{t_m}
\end{aligned}
\tag{10}
$$

where V_{t_m} and e_{t_m} are defined above. The state-space form (10) allows us to account for and incorporate all the information about the missing observables contained in the auxiliary variables.

The choice of modeling X_{t_m} as solely dependent on the observables Y_{t_m}, rather than depending in a more general way from the states s_{t_m}, is motivated by the fact that we want the auxiliary variables to be relevant *only* in real time, but we do not want them to affect the inference about the history of the latent states and shocks.

3   Design of the Forecasting Exercise

We present an application of the methodology described above to the estimated medium-scale model presented in Galí, Smets and Wouters (2012; henceforth GSW), which reformulates the well known Smets-Wouters (2007; henceforth SW) framework by embedding the theory of unemployment proposed in Galí (2011a,b). The main difference of the GSW with respect to the SW is the explicit introduction of unemployment, and the use of a utility specification that parameterizes wealth effects, along the lines of Jaimovich and Rebelo (2009). We present the main log-linearized equations of the model in Appendix A and refer to Galí, Smets and Wouters (2012) for an in depth discussion of the model. With respect to the GSW model, the only difference we introduce is that we remove the MA terms from the price and wage mark-up shocks to ensure diagonalizability.

The model is estimated on 8 data series for the US: GDP growth, consumption growth, investment growth, a measure of real wage inflation based on compensation per employee, the GDP deflator inflation, per capita employment, the nominal interest rate and the unemployment rate[5]. The first 5 variables are available at a quarterly frequency only, while the employment, unemployment and the interest rate are available at monthly frequency. The model however is specified and estimated at quarterly frequency: we report priors, modes and means of the estimated parameters in Appendix C. We will show how to derive the monthly specification for the model in order to take advantage of the additional available information, both on the observables and other timely macroeconomic variables.

The log-linear solution of the GSW has the form:

$$
\begin{aligned}
s_{t_q} &= \mathcal{T}_\theta s_{t_q-1} + \mathcal{B}_\theta \varepsilon_{t_q} \\
Y_{t_q} &= \mathcal{M}_{0,\theta} s_{t_q} + \mathcal{M}_{1,\theta} s_{t_q-1}.
\end{aligned}
\tag{11}
$$

where s_{t_q} is a vector of states of the models that describe the dynamics of the economy, including all the 8 structural shocks (risk premium, monetary policy, exogenous spending,

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[5]For robustness, we experimented also with the version of the Galí, Smets and Wouters that is based on two imperfectly measured wage concepts and obtain very similar results.

investment-specific technology shock, neutral technology, price mark-up, wage mark-up and exogenous labour supply) and the lags of the four variable that enter the observable equation in differences. Notice that it is equivalent 1) to define the differences in the state vector and set $\mathcal{M}_{0,\theta} = I_{k,n}$, where $I_{k,n}$ is a matrix of zeros and ones that just selects the appropriate rows of s_{t_q} or 2) to add the lags rather than the differences to the state equation and define the difference with the matrices $\mathcal{M}_{0,\theta}$ and $\mathcal{M}_{1,\theta}$. The advantage of 2) is that it is easier to compute the cube root, so we will follow this strategy.

We first verify that $\mathcal{T}_\theta$ in (11) can be diagonalized. Indeed it can, so we obtain the matrix D of eigenvalues and the corresponding matrix V of eigenvectors that satisfy $\mathcal{T}_\theta = VDV^{-1}$. We identify the model's real-valued cube root as described in the previous Section and we also verified that it is indeed the one that maximizes the likelihood.

We then produce the nowcast with the monthly model with and without auxiliary variables and compare it to the SPF's forecasts and to the forecast produced with the quarterly model, in which the last data point available is inputed for the higher frequency variables, as is generally done in policy institutions. And we will also obtain real-time estimates of purely model-based concepts like the output gap. The exercise is performed in real time: at each point of the forecast horizon we use the vintage of data that was available at the time.

As we will show in the next section, simply taking advantage of all the information available about the observables at a monthly frequency increases greatly the forecasting performance of the model. Incorporating information from key macro variables that are more timely also helps, especially for GDP growth. We consider a set of 15 macro and financial variables that are monitored more closely by professional and institutional forecasters[6]. These include measures of production (such as industrial production, capacity utilization, total construction, etc...), price data (CPI, various measures of PPI), financial market variables (the fed funds rate and the BAA-AAA spread), labour market variables, a housing market indicator, and various national accounts quantities. A full list and description of these series is listed in Table 4 in Appendix D, which describes a stylized calendar of data releases where the variables have been grouped in 32 clusters according to their timeliness. The stylization consists in associating a date with a group of variables with similar economic content (soft, quantities, prices and so on). This is a quite realistic representation of the calendar and will allow us to evaluate the changes in the forecast with variables with a given economic content. In the first column we indicate the progressive number associated to each "vintage" or release cluster, in the second column the data release, in the third the series and in the fourth the date the release refers to, which gives us the information on the publication lag. We can see, for example, that the Philadelphia Fed Survey is the first release referring to the current month m and it is published third Thursday of each month. Hard data arrive later. For example, the first release of industrial production regarding this quarter is published in the middle of the *second* month of the quarter. GDP, released the last week of the first month of the quarter refers to the previous quarter.

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[6]For a discussion of alternative ways of selecting the auxiliary variables, see Cervena and Schneider (2014), who apply the methodology proposed in the earlier version of this paper (Giannone, Monti and Reichlin, 2010) to a medium-scale DSGE model for Austria and address the issue of variable selection by proposing three different methodologies for the subsample selection.
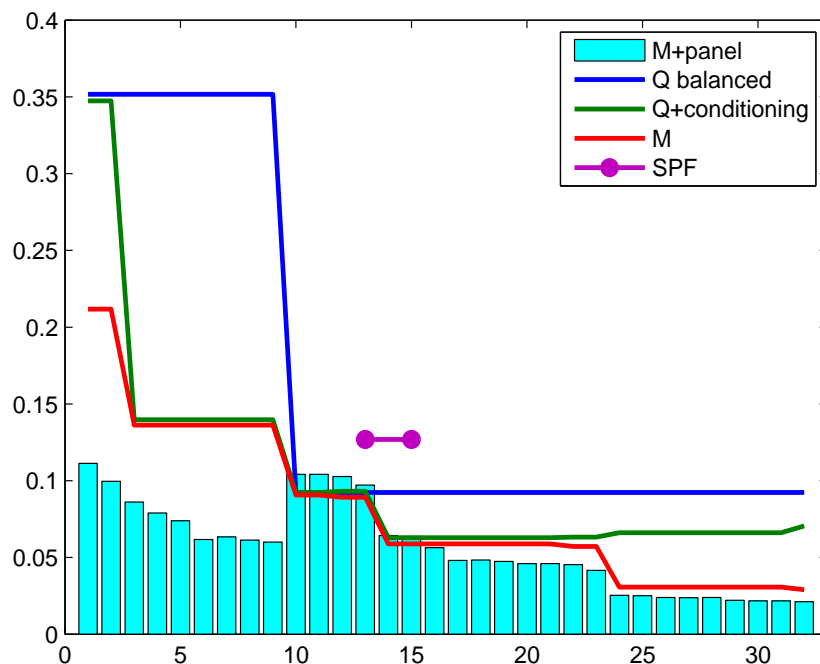
Figure 1: **MSFE of the unemployment nowcasts throughout the quarter**

4   Empirical results

In what follows we present nowcasts of quarterly GDP growth, the unemployment rate and
the output gap as defined by Galí et al. (2012). The model is estimated only once at the
beginning of the forecast evaluation window, *i.e.* using the data available at the end of the
fourth quarter of 2006. The exercise is performed in real time: at each point of the forecast
horizon we use the vintage of data that was available at the time. The forecast evaluation
sample spans from the first quarter 2007 to the first quarter of 2013. We will compare the
forecasting performance of four different models: the quarterly DSGE model based on the
balanced panel (Q balanced), the quarterly model in which we include the latest data point
for the monthly variables (Q+conditioning), the monthly model (M model) and the monthly
model augmented by auxiliary variables (M+panel), and, when possible, we benchmark them
against the forecasts the survey of professional forecasters (SPFs). We will analyse both point
forecasts and density forecasts.

The forecasts will be updated thirty-two times throughout the quarter, corresponding to
the stylized calendar 4. In this way we can associate to each update a date and a set of vari-
ables. The horizontal axis of the Figures below, indicate the grouping of releases correspond-
ing to the calendar. For example, referring to the axes of Figures 1-3, clusters 3, 14, and 24
correspond to the release of the Employment situation in each of the three months of the
quarter, release 10 corresponds to the flash estimate of GDP for the previous quarter and 12,
22 and 32 correspond to the last day of each month, where we account for the financial data.
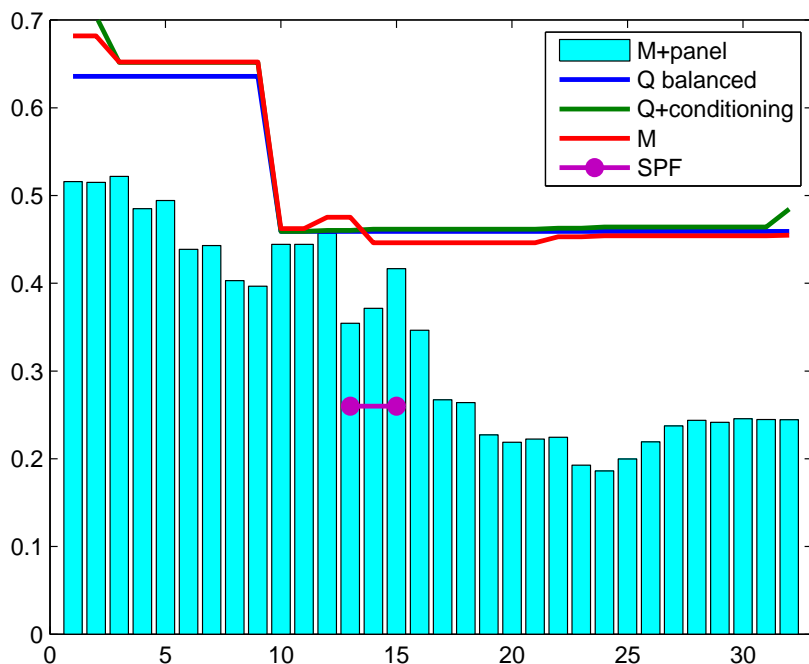
Figure 2: **MSFE of GDP growth nowcasts throughout the quarter evaluated against the "final estimate"**. We take the "final estimate", *i.e.* the release available after 2 quarters, as actual data when evaluating the performance of the model in real time

4.1 Point Forecasts

We first present the results for the point nowcast. Figures 1-3 report the mean square forecast error (MSFE) for the quarterly model (Q balanced, the blue line), the quarterly model conditioned on the available higher frequency observables (Q+cond., the green line), the monthly model (M, the red line) and the monthly model augmented with the auxiliary information (M+panel, the cyan bars). Where available, we also report a traditionally tough benchmark, the nowcast of the survey of professional forecasters' nowcast (SPF, in pink). We display it only from cluster 13 to cluster 15, that is around the beginning of the second month of the quarter when the SPF's forecasts are made, in order to align the SPF's and the models' information sets as closely as possible. Since the output gap is unobserved, we take it's ex-post estimate - *i.e.* the estimate produced by the quarterly DSGE model using all available data up to 2013Q1 - to be the "true" one, and we construct the MSFE of the nowcast produced by the alternative models we are consider with respect to it.

The nowcast of the quarterly model that uses the balanced panel (Q balanced) can be updated only once in the quarter, when the GDP for the past quarter is released (cluster 10). The nowcasts of the monthly model (M model) and of the quarterly model that accounts for the latest data point available for the monthly variables (Q+conditioning) are updated 6 times throughout the quarter, namely at each release of the monthly variables - the employment variables (3, 14, and 24) and the nominal interest rate (12, 22 and 32). The monthly model augmented by auxiliary variables (M+panel) is updated at each new release. The amount of jumps in the mean square forecast errors (MSFE) of each of the nowcasts in Figures 1-3
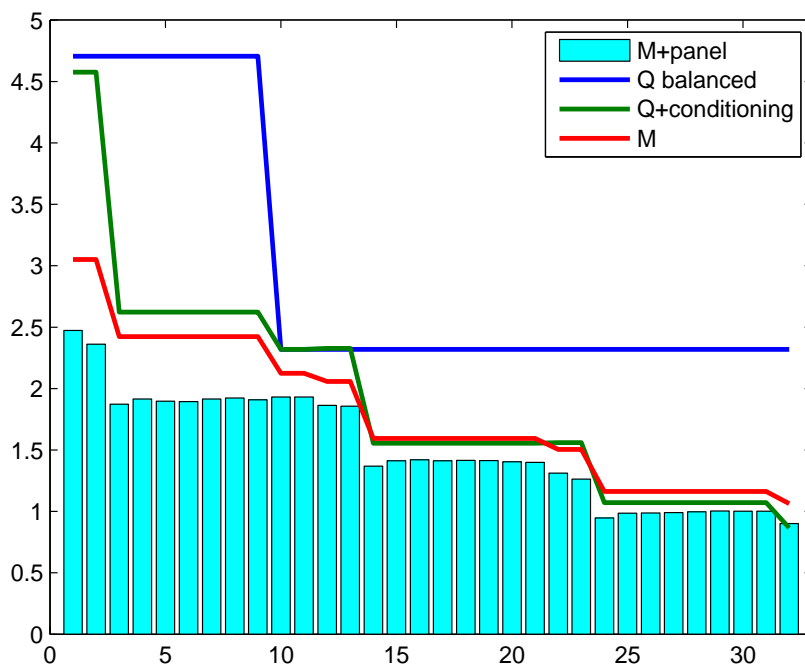
Figure 3: **MSFE of estimate of the current output gap throughout the quarter**

reflects how many times the nowcast is updated throughout the quarter.

Results indicate that the monthly specification is very useful especially when the focus is on a variable available at the monthly level such as unemployment (Figure 1). In this case the main advantage comes from the ability to account for the monthly observables in a more consistent way rather than from the real-time data flow, which only helps at the beginning of the quarter (clusters 1 to 9). The same is true for the output gap, which is defined as the difference between actual output and the output that would prevail in the flexible price and wage economy in absence of distorting price and wage markup shocks. In the GSW model, the output gap is very closely aligned to the total employment series, which is also available monthly and displays a similar MSFE profile. Figure 2 reports the evolution throughout the quarter of the MSFE of the nowcast of real GDP growth, respectively evaluated against the "final estimate" (*i.e.* the release available after 2 quarters). The monthly data flow is important in obtaining a more accurate forecast, though the increase in volatility coming from the auxiliary variables is, at times, damaging. This can also be inferred by the results for the density forecasts below in Figures 6 and 7. The reduction in the forecasting performance of the monthly model augmented by the auxiliary variables at information cluster 10, when GDP is released, is due to the fact that we link the auxiliary variables to the observables rather than the states of the model (see equation 9). We do so in order not to affect the inference about the history of the latent states and shocks, but the consequence is that when the information sets for each model are more closely aligned the forecast produced by each model will converge - and, when the information is the same, collapse - to the original model's forecast.

Tables (1)and (2) report the MSFE of nowcasts and forecasts up to 4 quarters ahead, for GDP growth and and the unemployment rate. We compare the forecasts produced by the

Figure 4: **The nowcast of the unemployment rate for 4 representative vintages**. The first three vintages correspond to the day of the release of employment data, which happens the first Friday of each month. The lower right panel correspond to the last day of the quarter.
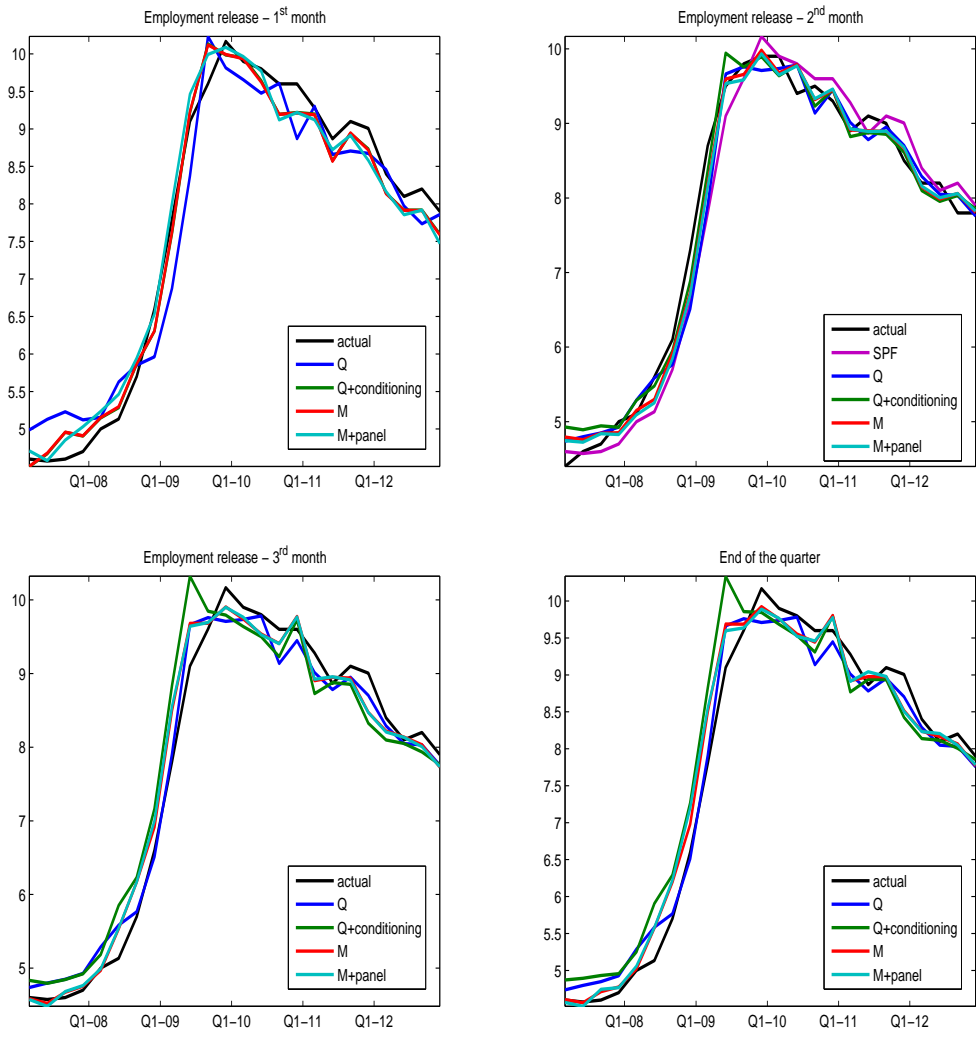
Figure 5: **The nowcast of GDP growth for 4 representative vintages**. The first three vintages correspond to the day of the release of employment data, which happens the first Friday of each month. The lower right panel correspond to the last day of the quarter.
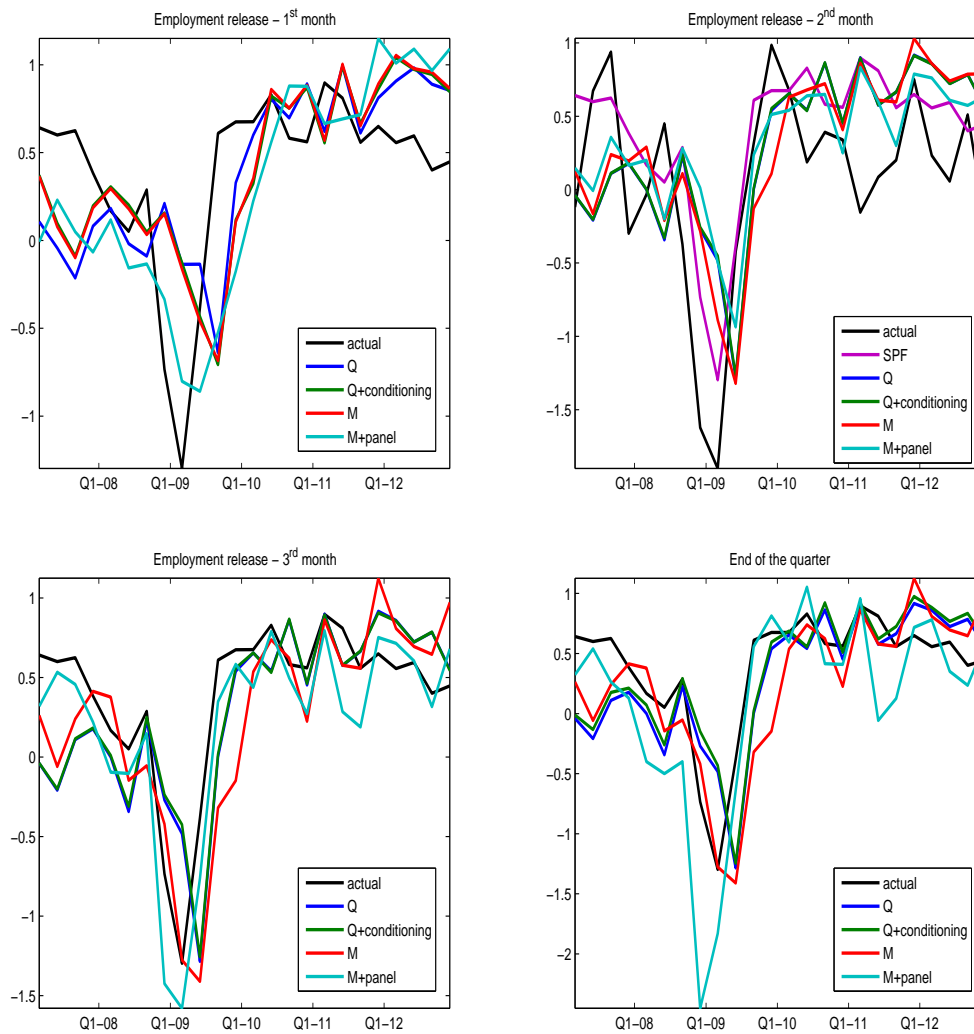
Table 1: MSFE of quarter-on-quarter GDP growth forecasts with horizons 0 to 4, produced on the first Friday of the second month of the quarter.

	SPF	Q	Q+cond	M	M+panel
Q0	0.2599	0.4592	0.4617	0.4461	0.4166
Q1	0.4925	0.6634	0.6644	0.6890	0.7019
Q2	0.7170	0.6985	0.6993	0.7210	0.7225
Q3	0.9206	0.7052	0.7054	0.7222	0.7242
Q4	0.9640	0.7985	0.7984	0.8114	0.8086

Table 2: MSFE of the unemployment rate forecasts with horizons 0 to 4, produced on the first Friday of the second month of the quarter.

	SPF	Q	Q+cond	M	M+panel
Q0	0.1268	0.0924	0.0629	0.0588	0.0637
Q1	0.4096	0.3763	0.2951	0.2934	0.3273
Q2	0.0682	0.7972	0.7387	0.6370	0.6473
Q3	1.7669	1.3293	1.2537	1.2931	1.4581
Q4	2.9270	1.9859	1.9222	1.9985	2.2456

In order to avoid giving an undue advantage to the models that account for the real time data flow, when comparing them to the SPF, we show the forecasts that would be produced by all our models on the day of the release of the employment data on the first Friday of the second month of the quarter, *i.e.* around the time the SPF forecasts are produced (information Cluster 14).

SPF, the quarterly DSGE model (Q), the the monthly DSGE model (M) and the monthly DSGE model that also exploits the information contained in the panel (M+panel). Hence, in order to match the information available to them at the time of the forecast, we generate the forecasts of tables (1)and (2) with information Cluster 14, which corresponds to the release of the Employment data on the first Friday of the second month of the quarter.

Two features of the results are quite striking. First, the SPF's forecasting performance in our evaluation sample 2007Q1- 2013Q1 is much better than any of the model-based ones for GDP growth and but much worse for the unemployment rate. This result however is very sample dependent. When looking at the pre-crisis sample, the SPF perform at least as well as the quarterly model also for unemployment, as is expected from this traditionally tough to beat benchmark. Second, as the forecasting horizon increases, the performance of the Q, Q+condititoning, M and M+panel models becomes more and more similar. This is due to the way we choose to link the auxiliary to the observables rather than the state variables directly, because this modeling choice implies that the information that can be extracted for the panel of variables is relevant only when nowcasting.

Figures 4 and 5 report the nowcast for the employment rate and GDP growth for four representative vintages, namely the day of each month in which the employment data is released and the last day of the quarter, when all information available in the quarter in revealed.

4.2 Density Forecasts

We are also interested in characterising and evaluating the uncertainty associated with the predictions of the model: we do so computing the predictive density of the models and the
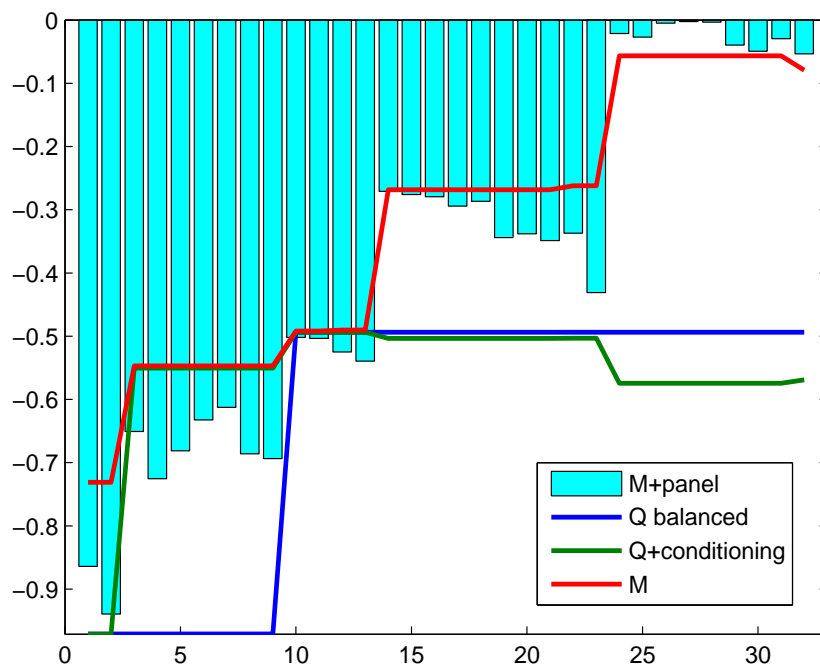
Figure 6: **Log predictive score of the nowcast of unemployment after different releases**

associated log predictive scores. The log predictive score is a widely used scoring rule, used to evaluate the quality of probabilistic forecasts given a set of outcomes. Formally it is defined here as:

$$S_h(\mathcal{M}) = \frac{1}{N_h} \sum_{t=T}^{T+N_h-1} \ln p(y_{t+h}|Y_{1:T-1}, \mathcal{M}), \tag{12}$$

where h is the forecast horizon, T is the beginning of the forecast horizon and $\ln p(y_{t+h}|Y_{1:T-1}, \mathcal{M})$ is the marginal likelihood for $h = 1$.

Figures 6 and 7 report the log predictive score of the nowcast of unemployment and GDP growth produced after each of the 32 clusters of releases. Interestingly, in both cases the monthly model seems overall to be the best performing model. In the case of unemployment, the monthly model that exploits the panel ranks very close to it at most "vintages." In the case of GDP growth, the monthly model that exploits the panel ranks worse, probably reflecting the excessive volatility introduced by the panel in the evaluation sample and which can also be seen in Figure 5, especially in the bottom right panel, which corresponds to cluster 32.

5   Conclusions

This paper shows how to obtain a mapping from a quarterly DSGE model to its real stable monthly counterpart. We use this technique to derive the monthly counterpart of the Galí *et al.* (2012) model. We then augment it by auxiliary macro indicators which, because of their timeliness, can be used to obtain a nowcast of the structural model. We show empirical results for the quarterly growth rate of GDP, the monthly unemployment rate and welfare
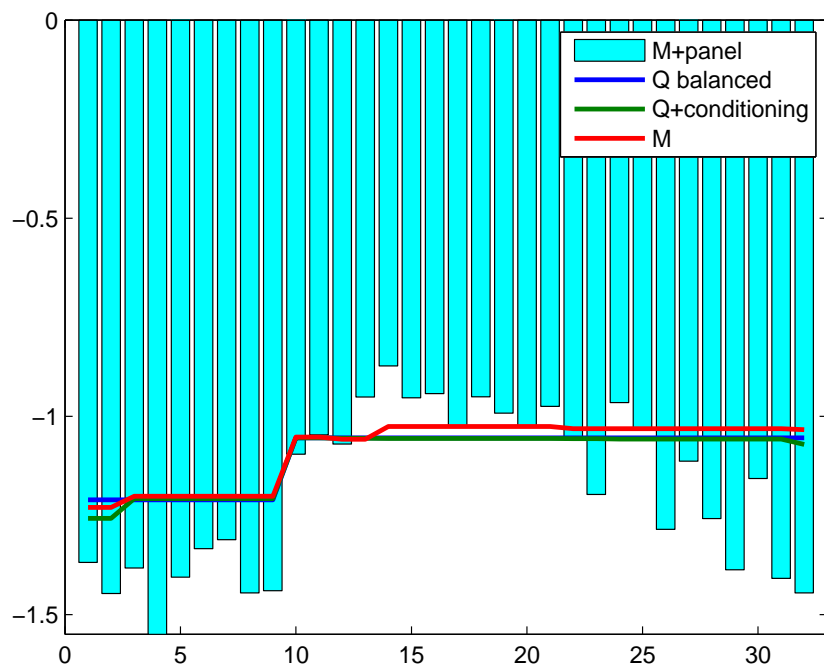
Figure 7: **Log predictive score of the nowcast of GDP growth after different releases**

relevant output gap defined in Galí *et al.* as the distance between the actual and the natural level of output. Results show that the augmented monthly model does best for nowcasting.

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Appendix A

Here we summarize the key log-linear equations of the GSW model. We refer to Galí, Smets and Wouters (2012) for a more detailed description of the model.

- Consumption Euler equation:

$$\hat{c}_t = c_1 E_t \left[\hat{c}_{t+1}\right] + (1 - c_1)\hat{c}_{t-1} - c_2 \left(\hat{R}_t - E_t \left[\hat{\pi}_{t+1}\right] - \hat{\varepsilon}_t^b\right)$$

with $c_1 = 1 = /(1 + h)$, $c_2 = c_1(1 - h)$ where h is the external habit parameter. $\hat{\varepsilon}_t^b$ is the exogenous AR(1) risk premium process.

- Investment Euler equation:

$$\hat{i}_t = i_1\hat{i}_{t-1} + (1 - i_1)\hat{i}_{t+1} + i_2\hat{Q}_t^k + \hat{\varepsilon}_t^q$$

with $i_1 = 1/(1+\beta)$, $i_2 = i_1/\Psi$ where β is the discount factor and Ψ is the elasticity of the capital adjustment cost function. $\hat{\varepsilon}_t^q$ is the exogenous AR(1) process for the investment specific technology.

- Value of the capital stock:

$$\hat{Q}_t^k = -\left(\hat{R}_t - E_t \left[\hat{\pi}_{t+1}\right] - \hat{\varepsilon}_t^b\right) + q_1 E_t \left[\hat{r}_{t+1}^k\right] + (1 - q_1)E_t \left[\hat{Q}_{t+1}^k\right]$$

with $q_1 = \hat{r}_*^k/(\hat{r}_*^k + (1 - \delta))$ where $\hat{r}_*^k$ is the steady state rental rate to capital, and δ the depreciation rate.

- Aggregate demand equals aggregate supply:

$$\hat{y}_t = \frac{c_*}{y_*}\hat{c}_t + \frac{i_*}{y_*}\hat{i}_t + \hat{\varepsilon}_t^g + \frac{r_*^k k_*}{y_*}\hat{u}_t \tag{13}$$

$$= \mathcal{M}_p \left(\alpha\hat{k}_t + (1 - \alpha)\hat{L}_t + \hat{\varepsilon}_t^a\right) \tag{14}$$

with $\mathcal{M}_p$ reflecting the fixed costs in production which corresponds to the price markup in steady state. $\hat{\varepsilon}_t^g, \hat{\varepsilon}_t^a$ are the AR(1) processes representing exogenous demand components and the TFP process.

- Price-setting under the Calvo model with indexation:

$$\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \pi_1 \left(E_t \left[\hat{\pi}_{t+1}\right] - \gamma_p\hat{\pi}_t\right) - \pi_2\mu_t^p + \hat{\varepsilon}_t^p$$

with $\pi_1 = \beta$, $\pi_2 = (1 - \theta_p\beta)(1 - \theta_p)/\left[\theta_p(1 + (\mathcal{M}_p - 1)\varepsilon_p)\right]$ and θ_p and γ_p are, respectively, the probability and indexation of the Calvo model, and ε_p the is curvature of the aggregator function. The price markup μ_t^p is equal to the inverse of the real marginal $\hat{mc}_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}k_t - \hat{A}_t$.

- Wage-setting under the Calvo model with indexation:

$$\pi_t^w = \gamma_w\pi_{t-1}^p + \beta E_t \left[\pi_{t+1}^w - \gamma_w\pi_t^p\right] - \lambda_w\phi u_t + \lambda_w\mu_t^w$$

where the unemployment rate $u_t = l_t - n_t$ is defined so as to include all the individuals who would like to be working (given current labour market conditions, and while internalizing the benefits that this will bring to their households) but are not currently employed.

- Capital accumulation equation:

$$\hat{\bar{k}}_t = \kappa_1 \hat{\bar{k}}_{t-1} + (1 - \kappa_1)\hat{i}_t + \kappa_2 \hat{\varepsilon}_t^q$$

with $\kappa_1 = 1 - (i_*/\bar{k}_*)$, $\kappa_2 = (i_*/\bar{k}_*)(1 + \beta)\Psi$. Capital services used in production is defined as: $\hat{k}_t = \hat{u}t + \hat{\bar{k}}_{t-1}$

- Optimal capital utilisation condition:

$$\hat{u}_t = \frac{1 - \phi}{\phi}\hat{r}_t^k$$

with ϕ being the elasticity of the capital utilisation cost function.

- Optimal capital/labour input condition:

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t$$

- Monetary policy rule:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)(r_\pi \hat{\pi}_t + r_y ygap_t) + r_{\Delta y}\Delta y_t + \varepsilon_t^r$$

where $ygap_t = y_t - yflex_t$ is the difference between actual output and the output in the flexible price and wage economy in absence of distorting price and wage markup shocks. The following parameters are not identified by the estimation procedure and therefore calibrated: $\delta = 0.025$ and $\varepsilon_p = 10$.

Appendix B

As an illustration we use a simple New-Keynesian dynamic stochastic general equilibrium model, as the one used for example in Del Negro and Schorfheide (2004). The only source of nominal rigidities in this model is the presence of adjustment costs that firms incur in when changing their prices.

The log-linearized system can be reduced to three equations in output inflation and the interest rate:

$$
\begin{aligned}
\hat{y}_t - \hat{g}_t &= E_t \left(\hat{y}_{t+1} - \hat{g}_{t+1} \right) - \frac{1}{\tau} \left(\hat{r}_t - E_t \hat{\pi}_{t+1} - \rho_z \hat{z}_t \right) \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa (\hat{y}_t - \hat{g}_t) \\
\hat{r}_t &= \psi_1 (1 - \rho_r) \hat{\pi}_t + \psi_2 (1 - \rho_r) \hat{y}_t + \rho_r \hat{r}_{t-1} + \varepsilon_{r,t}
\end{aligned}
\tag{15}
$$

The first equation, often referred to as New Keynesian IS curve, is an intertemporal Euler equation for consumption in terms of output. The second equation is the familiar New-Keynesian Phillips curve and the last equation is a standard Taylor rule. The shock to the technology process $\hat{z}_t$, which is assumed to evolve following the process:

$$
\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_t^z.
$$

Also the government spending shock follows an AR(1) process:

$$
\hat{g}_{t+1} = \rho_g \hat{g}_t + \varepsilon_t^g.
$$

The relation between log-deviations from steady state and observable output growth, CPI inflation and the annualized nominal interest rate is given by the following measurement equation.

$$
\begin{aligned}
INFL_t &= \pi^* + 4\hat{\pi}_t \\
RA_t &= \pi^* + r^* + 4\hat{r}_t \\
\Delta \ln GDP_t &= \ln \gamma + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t
\end{aligned}
\tag{16}
$$

The model given by equations (15) can then solved with standard techniques, such as those proposed by Sims (2002) among others. More specifically, the model has a solution in terms of:

$$
\begin{aligned}
s_t &= A_\theta s_{t-1} + B_\theta \varepsilon_t \\
Y_t &= C(L) s_t
\end{aligned}
\tag{17}
$$

where $s_t = [\hat{g}_t, \ \hat{z}_t, \ \hat{r}_t, \ \hat{y}_t, \ \hat{\pi}_t]'$, $Y_t = [INFL_t, \ RA_t, \ \Delta \ln GDP_t]'$ and

$$
A_\theta = \begin{bmatrix}
\rho_g & 0 & 0 & 0 & 0 \\
0 & \rho_z & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & 0 & 0 \\
a_{41} & a_{42} & a_{43} & 0 & 0 \\
a_{51} & a_{52} & a_{53} & 0 & 0
\end{bmatrix}
$$

where $a_{31}, \ ..., \ a_{53}$ are functions of the parameters in (15). We obtain the eigenvalues and eigenvectors of this matrix as the solution to the equation $A_\theta V = VD$, where D a diagonal

matrix and its diagonal entries are the eigenvalues of A_θ, and V is the matrix of eigenvectors of A_θ. A_θ is diagonalizable, *i.e.* $A_\theta = VDV^{-1}$, if and only if V is invertible. Because A_θ is a triangular matrix, its eigenvalues are easily determined as its diagonal entries. D and V have the form:

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_g & 0 & 0 \\ 0 & 0 & 0 & \rho_z & 0 \\ 0 & 0 & 0 & 0 & a_{33} \end{bmatrix} \quad V = \begin{bmatrix} 0 & 0 & \frac{\rho_g(\rho_g - a_{33})}{\rho_g a_{51} - a_{33}a_{51} + a_{31}a_{53}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho_z(\rho_z - a_{33})}{\rho_z a_{52} - a_{33}a_{52} + a_{32}a_{53}} & 0 \\ 0 & 0 & \frac{\rho_g a_{31}}{\rho_g a_{51} - a_{33}a_{51} + a_{31}a_{53}} & \frac{\rho_z a_{32}}{\rho_z a_{52} - a_{33}a_{52} + a_{32}a_{53}} & \frac{a_{33}}{a_{53}} \\ 0 & 1 & -\frac{-\rho_g a_{41} + a_{33}a_{41} - a_{31}a_{43}}{\rho_g a_{51} - a_{33}a_{51} + a_{31}a_{53}} & -\frac{-\rho_z a_{42} + a_{33}a_{42} - a_{32}a_{43}}{\rho_z a_{52} - a_{33}a_{52} + a_{32}a_{53}} & \frac{a_{43}}{a_{53}} \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Having two zero eigenvalues, A_θ is clearly singular[7], but its zeros eigenvalues are semi-simple, that is, they are associated to linearly independent eigenvectors.Therefore this matrix is in general diagonalizable, unless any of the eigenvectors associated with the non-zeros eigenvalues are linearly dependent.

It is possible to characterize the all the cube roots of D and consequently of $\mathcal{T}_\theta$: we can then verify which of the latter, if any, have real coefficients, and which among the real ones delivers the highest likelihood. The diagonal entries of the matrix D are the eigenvalues of this system. The cube roots can be obtained as $\mathcal{T}_\theta = VD^{\frac{1}{3}}V^{-1}$. Since D is diagonal its cube roots can be determined simply by taking the cube roots of the diagonal entries. Any real or complex number λ has 3 cube roots and they can be characterized as follows:

$$\sqrt[3]{\lambda} = \begin{cases} r \\ r(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\ r(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) \end{cases}$$

Therefore the diagonal $n \times n$ matrix D will have at most 3^n cube roots, obtained as combinations of the 3 cube roots corresponding to each eigenvalue. For this relatively small-scale example, we characterize all the $3^3 = 27$ cube roots of this matrix and verify that there is effectively just one that has real coefficients and it is the that has only real-valued eigenvalues. In the case of larger matrices such as the one of the GSW model (which has $3^{16} = 43046721$ cube roots), characterizing all the roots my be computationally very burdensome, but we can always randomly draw from the set of all cube roots and verify whether it has real coefficients, and if so evaluate its likelihood.

---

[7]In principle, the state equation could be simplified further to make A_θ full rank, but it is instructive in this example to maintain the singularity of A_θ and show that it does not prevent the matrix from being diagonalizable.

	Prior Distribution			Posterior Distribution	
	Distr.	mean	st.dev	mode	mean
h	N	0.7	0.1	0.67	0.65
Ψ	N	4.0	0.25	4.59	4.61
$\mathcal{M}_p$	N	1.25	0.12	1.84	1.80
$\mathcal{M}_w$	N	1.25	0.25	1.01	1.03
ν	B	0.5	0.2	0.15	0.17
r_π	N	1.5	0.125	1.84	1.83
r_{ygap}	N	0.12	0.05	0.08	0.76
$r_{\Delta ygap}$	N	0.12	0.05	0.14	0.13
θ_w	B	0.5	0.1	0.50	0.54
θ_p	B	0.5	0.1	0.73	0.76
γ_p	B	0.5	0.1	0.37	0.40
γ_w	B	0.5	0.1	0.25	0.29
ψ	B	0.5	0.15	0.49	0.44
ρ_r	B	0.75	0.10	0.84	0.82
ϕ	N	2	0.5	4.44	0.43
τ	N	4	0.1	0.43	0.42
ρ_b	B	0.5	0.2	0.16	0.18
ρ_q	B	0.5	0.2	0.79	0.78
ρ_g	B	0.5	0.2	0.97	0.97
ρ_a	B	0.5	0.2	0.97	0.97
ρ_{ms}	B	0.5	0.2	0.07	0.1
σ_b	U	2.5	1.44	2.64	2.5
σ_q	U	2.5	1.44	0.52	0.53
σ_g	U	2.5	1.44	0.46	0.47
σ_a	U	2.5	1.44	0.42	0.42
σ_p	U	2.5	1.44	1.77	1.97
σ_r	U	2.5	1.44	0.23	0.24
σ_w	U	2.5	1.44	2.53	2.43
σ_χ	U	2.5	1.44	1.45	1.47

Table 3: Prior and posterior distribution of the parameters of the model estimated over the period 1959Q2 to 2006Q4. The remains parameters are calibrated to their means in GSW (2012)

	timing	release	publication lag
1	1st day of the 1st month of the quarter	-	-
2	1st business day of the 1st month of the qtr	PMI and construction	m-1, m-2
3	1st Friday of the 1st month of the qtr	Employment situation	m-1
4	Middle of the 1st month of the qtr	PPI	m-1
5	Middle of the 1st month of the qtr	Inventories, Sales	m-2
6	15th to 17th of the 1st month of the qtr	Industrial Production and Capacity Utilization	m-1
7	Middle of the 1st month of the qtr	CPI	m-1
8	3rd Thursday of the 1st month of the qtr	Business Outlook Survey: Philadelphia Fed	m
9	later part of the 1st month of the qtr	housing starts	m-1
10	Last week of 1st month of the qtr	GDP release	q-1
11	Day after GDP release	PCE, RDPI	m-1
12	Last day of the 1st month of the qtr	Fed Funds rate	m
13	1st business day of the 2nd month of the qtr	PMI and construction	m-1, m-2
14	1st Friday of the 2nd month of the qtr	Employment situation	m-1
15	Middle of the 2nd month of the qtr	PPI	m-1
16	Middle of the 2nd month of the qtr	Inventories, Sales	m-2
17	15th to 17th of the 2nd month of the qtr	Industrial Production and Capacity Utilization	m-1
18	Middle of the 1st month of the qtr	CPI	m-1
19	3rd Thursday of the 2nd month of the qtr	Business Outlook Survey: Philadelphia Fed	m
20	later part of the 2nd month of the qtr	housing starts	m-1
21	End of the 2nd month of the quarter	PCE, RDPI	m-1
22	Last day of the 3rd month of the qtr	Fed Funds rate	m
23	1st business day of the 3rd month of the qtr	PMI and construction	m-1, m-2
24	1st Friday of the 3rd month of the qtr	Employment situation	m-1
25	Middle of the 3rd month of the qtr	PPI	m-1
26	Middle of the 3rd month of the qtr	Inventories, Sales	m-2
27	15th to 17th of the 3rd month of the qtr	Industrial Production and Capacity Utilization	m-1
28	Middle of the 3rd month of the qtr	CPI	m-1
29	3rd Thursday of the 3rd month of the qtr	Business Outlook Survey: Philadelphia Fed	m
30	later part of the 3rd month of the qtr	housing starts	m-1
31	End of the 3rd month of the quarter	PCE, RDPI	m-1
32	Last day of the 3rd month of the qtr	Fed Funds rate	m

Table 4: Data releases are indicated in rows. Column 1 indicates the progressive number associated to each "vintage." Column 2 indicates the official dates of the publication. Column 3 indicates the releases. Column 4 indicates the publishing lag: e.g. IP is release with 1-month delay (m-1).